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Universal low energy features of two-body systems

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Abstract We apply renormalization ideas to study low-energy interactions in two-body systems. As we will see this method highlights a model-independent description of a broad variety of systems ranging from ultra-could atoms to NN and $\Lambda\Lambda$ interactions.

Keywords Low energy scattering · Atom-atom · Baryon-baryon

The renormalization with boundary conditions (BC) are best illustrated by the effective range expansion where the S-wave phase shift is given by (p is CM momentum)

$$p\cot\delta_0(p) = -\frac{1}{\alpha_0} + \frac{1}{2} r_0 p^2 + \dots \tag{1}$$

where r_0 is the effective range and α_0 the scattering length. Using the superposition principle one obtains a bilinear relation dubbed as *universal low energy theorem* [1,2],

$$r_0 = A + \frac{B}{\alpha_0} + \frac{C}{\alpha_0^2}\,, \tag{2}$$

where the coefficients A, B and C depend *only* on the long distance potential. This way $V(r)$ and α_0 are regarded as *independent* variables. We present below a variety of situations.

The case of ultra-could collisions between neutral atoms described by the van der Waals (vdW) potential $V(r) = -\sum_{n=6}^{\infty} \frac{C_n}{r^n}$ where C_n are the vdW coefficients was analyzed in Refs. [3,4]. The terms $n = 6, 8, 10$ are usually retained and the singularity appearing in the vdW potential led to the use of phenomenological potentials, like the Lennard-Jones, modeling short-distance physics. The correlation, Eq. (2), is plotted in Fig. 1 (upper left panel) for the term $n = 6$ in units of the vdW radius $R_6 = (2\mu C_6)^{1/4}$ where the coefficients are

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BB	A [fm]	B [fm^2]	C [fm^3]	α_0[fm]	r_0[fm]	$\alpha_0^{\rm exp}$[fm]	$r_0^{\rm exp}$[fm]
np	2.437	-5.345	5.277	input	2.672	-23.74(2)	2.77(5)
pp(S)	2.465	-5.661	5.996	-17.806	2.802	-17.46	2.845
pp(C)	1.982	-4.985	7.009	-7.706	2.747	-7.8149(29)	2.769(14)
nn	2.467	-5.666	6.003	-19.626	2.771	-18.9(4)	2.75(11)

Table 1 Coefficients A,B and C of Eq. (2) using the OBE potential Eq. (3) implementing CSB and values for the LEPs using the short distance connection. We also list experimental or recommended values.

$A/R_6 = 1.39473$, $B/R_6 = -1.33333$ and $C/R_6 = 0.637318$, together with the experimental and potential model determinations of α_0 and r_0 known up to date and compiled in Ref. [3, 4]. On the light of this clear universality one realizes that the leading $1/r^6$ contribution suffices to accurately describe low energy atom-atom scattering with just two parameters in a wide energy range.

In our studies of the NN interaction we consider the OBE potential with exchange of π, σ, ρ, ω keeping the spin-flavor structure of the large-N_c limit according with Refs. [6,7] with the parameters fixed to the condition that the $^1S_0$ np phase shift be reproduced,

$$V_{^1S_0}(r) = -\frac{g_{\pi NN}^2 m_\pi^2}{16\pi M_N^2}\frac{e^{-m_\pi r}}{r} - \frac{g_{\sigma NN}^2}{4\pi}\frac{e^{-m_\sigma r}}{r} + \frac{{g_{\omega NN}}^2}{4\pi}\frac{e^{-m_\omega r}}{r} - \frac{{f_{\rho NN}}^2 m_\rho^2}{8\pi M_N^2}\frac{e^{-m_\omega r}}{r} \,. \qquad (3)$$

As was shown in Ref. [8] and more recently in Ref. [9] the unnaturally large scattering length in this channel triggers a fine-tuning problem in the potential parameters. However using renormalization one disentangles the unknown short-distance physics from the well established long-distance one by fixing from the start low-energy parameters (LEPs). In particular, one finds vector mesons to play a marginal role in the description of NN scatering below pion production threhold [8]. In addition one obtains $m_\sigma = 501(25)\,\mathrm{MeV}$, $g_{\sigma NN} = 9(1)$, $g_{\omega NN} \sim g_{\omega NN}^{SU(3)} = 3 g_{\rho NN}$ even when charge symmetry breaking is implemented by considering the neutron-proton and charged-neutral pion mass differences [9]. Using the more conventional regular boundary condition $u(0) = 0$ two very good but mutually incompatibles fits are obtained, two scenarios which correspond with the absence or presence of spurious bound states [8] and in any case with or without large $SU(3)$ violation of $g_{\omega NN}$ respectively.

Using the previous potential in Ref. [5] we have re-interpreted the old nuclear Wigner (spin-isospin) SU(4) symmetry as a *long distance symmetry*. This symmetry implies equal singlet and triplet potentials $V_{1S0} = V_{3S1}$, a fact that agrees with the large-N_c expectations. The renormalization approach allows a breaking of the symmetry at short distances having $\alpha_{0,1S0} \neq \alpha_{0,3S1}$ and $r_{0,1S0} \neq r_{0,3S1}$ even with the same long-distance potential. This can be clearly seen in Fig. 1 (upper right panel) where we plot Eq. (2) in the case of OPE and OPE+σ. The only difference between $r_{0,1S0}$ and $r_{0,3S1}$ resides in the numerical values of the scattering lengths $\alpha_{0,1S0}$ and $\alpha_{0,3S1}$ but the coefficients A,B and C in Eq. (2) are the same in both channels. The experimental values fall almost on top of the curve.

The posibility of connecting different NN isospin channels has been analyzed in Ref. [9] using a *short distance connection*. One is able to obtain a bilinear relation between the scattering lengths of different problems $\alpha_{0,2} = (a\ \alpha_{0,1} + b)/(c\ \alpha_{0,1} + d)$ where the coefficients a,b,c and d depend *only* on the long distance potential. Fixing the scattering length in one channel, e.g. $\alpha_{0,np} = -23.74\,\mathrm{fm}$, with the suitable incorporation of the Coulomb interaction $V_C(r) = \alpha/r$, with $\alpha \simeq 1/137$ for the pp(C) case, we obtain the rest of LEPs displayed in Table 1. In Fig. 1 (lower left panel) we have plotted Eq. (2) for the np system and the predicted LEPs for the other channels. This figure indicates that the strongly interacting component is close to a unique curve whereas adding the Coulomb interaction distortes the long distance

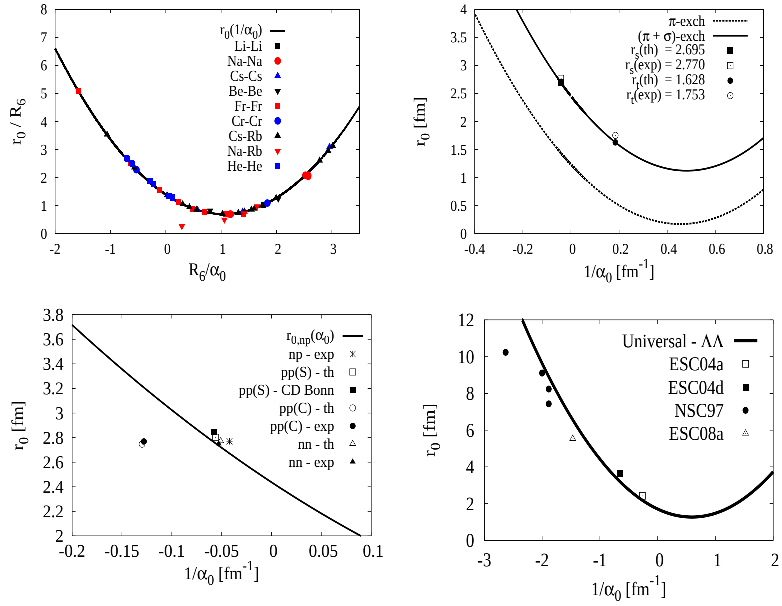

Fig. 1 Universal relations: (upper left panel) Atom-Atom vdW correlation in units of R_6, (upper right panel) NN $^1S_0$-$^3S_1$ Wigner correlation, (lower left panel) nn,np and pp(S) short-distance connection correlation and (lower right panel) $\Lambda\Lambda$ long distance correlation compared to potential models calculations (see e.g. [10]) .

potential and hence the pp(C) LEPs move away from the curve. No explicit $\rho - \omega$ mixing seems necessary as it is a short range effect which is reparameterized in terms of α_0.

Finally, we can assume systems with strangeness such as $\Lambda\Lambda$ for example (see e.g. [10] and references therein). If Λ-hyperons interact exchanging 1σ- and 1ω-mesons and we use SU(3) relations for the couplings $g_{\sigma\Lambda\Lambda} = g_{\omega NN}$ and $g_{\omega\Lambda\Lambda} = 2/3 g_{\omega NN}$ we obtain $A = 1.705\text{fm}$, $B = -1.475\text{fm}^2$ and $C = 1.245\text{fm}^3$. The universal curve for such a simple OBE potential is displayed in Fig. 1 (lower right panel) and is not far from more sophisticated potential model calculations [10].

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